

The Existence of Optical Solitons on Wavelength Division  
Multiplexed Beams in a Nonlinear Fiber

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*Simple analytic expression for the initial fundamental optical solitons on wavelength division multiplexed (WDM) beams in a nonlinear fiber has been found. For an ideal fiber with no loss and uniform group velocity dispersion (GVD) in the anomalous GVD region, the initial form is  $(1 + 2(M-1))^{-1/2} \text{sech}(\tau)$ , where  $M$  is the number of WDM beams and  $\tau$  is the normalized time. Computer simulation shows that these initial pulses on WDM beams in this fiber will propagate undistorted without change in their shapes for arbitrarily long distances. Discovery of the existence of solitons on WDM beams presents the ultimate goal for optical fiber communication on multiple wavelength beams in a single fiber.*

## I. Introduction

The discovery in 1973 that optical soliton [1] on a single wavelength beam can exist in fiber is one of the most significant event since the perfection of low-loss optical fiber communication. This means that, in principle, data pulses may be transmitted in a fiber without degradation forever. This soliton discovery sets the ultimate goal for optical fiber communication on a single wavelength beam.

Another most significant event is the development of wavelength division multiplexed (WDM) transmission in a single mode fiber [2]. This means that multiple beams of different wavelengths, each carrying its own data load, can propagate simultaneously in a single mode fiber. This WDM technique provides dramatic increase in the bandwidth of a fiber. However, due to the presence of complex nonlinear interaction between co-propagating pulses on different wavelength beams, it is no longer certain that WDM solitons can exist.

The existence of solitons is a blissful event in nature. It is a marvel that the delicate balance between the dispersion effect and the nonlinear effect can allow a specially shaped optical pulse to propagate in the fiber without degradation. This is called a temporal soliton [1]. It is an equal marvel that the delicate balance between the

diffraction effect and the nonlinear effect can also allow a specially shaped pulse to propagate in planar waveguide or array waveguides without degradation. This is called a spatial soliton [3]. They occur only on single wavelength beam.

When beams with different wavelengths co-propagate in a single mode fiber, such as in the wavelength division multiplexed (WDM) case [2], interaction of pulses on different beams via the nonlinear cross phase modulation (CPM) effect (the Kerr effect) is usually instrumental in destroying the integrity of solitons on these wavelength multiplexed beams. Other recent applications of CPM effect in fiber have been reported [4].

The purpose of this paper is to show that temporal solitons can exist on WDM beams in a single fiber under appropriate conditions. The existence of these solitons critically depends on the presence of the nonlinear cross phase modulation effect of the WDM beams. Just as the earlier single beam soliton case, this discovery sets the ultimate goal for optical fiber communication on WDM beams.

## II. The Fundamental Equations

The fundamental equations governing  $M$  numbers of co-propagating waves in a nonlinear fiber including the CPM

phenomenon are the coupled nonlinear Schrodinger equations [5]:

$$\begin{aligned} \frac{\partial A_j}{\partial z} + \frac{1}{v_{gj}} \frac{\partial A_j}{\partial t} + \frac{1}{2} \alpha_j A_j = \frac{1}{2} \beta_{2j} \frac{\partial^2 A_j}{\partial t^2} \\ - \gamma_j (|A_j|^2 + 2 \sum_{m \neq j}^M |A_m|^2) A_j \end{aligned}$$

(j = 1, 2, 3, ..... M ) (1)

Here, for the jth wave,  $A_j(z, t)$  is the slowly-varying amplitude of the wave,  $v_{gj}$ , the group velocity,  $\beta_{2j}$ , the dispersion coefficient (  $\beta_{2j} = dv_{gj}^{-1}/d\omega$  ),  $\alpha_j$ , the absorption coefficient, and

$$\gamma_j = \frac{n_2 \omega_j}{c A_{eff}} \quad (2)$$

is the nonlinear index coefficient with  $A_{eff}$  as the effective core area and  $n_2 = 3.2 \times 10^{-16} \text{ cm}^2/\text{W}$  for silica fibers,  $\omega_j$  is the carrier frequency of the jth wave,  $c$  is the speed of light, and  $z$  is the direction of propagation along the fiber.

Introducing the normalizing coefficients

$$\tau = \frac{t - (z/v_{g1})}{T_0}$$

$$d_{1j} = (v_{g1} - v_{gj}) / v_{g1} v_{gj}, \quad (3)$$

$$\xi = z/L_{D1},$$

$$L_{D1} = T_0^2/|\beta_{21}|,$$

and setting

$$u_j(\tau, \xi) = (A_j(z, t)/\sqrt{P_{0j}}) \exp(\alpha_j L_{D1} \xi/2) \quad (4)$$

$$L_{NLj} = 1/(\gamma_j P_{0j})$$

$$L_{Dj} = T_0^2/|\beta_{2j}| \quad (5)$$

gives

$$\begin{aligned} i \frac{\partial u_j}{\partial \xi} = & \frac{\operatorname{sgn}(\beta_{2j}) L_{D1}}{2 L_{Dj}} \frac{\partial^2 u_j}{\partial \tau^2} - i \frac{d_{1j}}{T_0} L_{D1} \frac{\partial u_j}{\partial \tau} \\ & - \frac{L_{D1}}{L_{NLj}} \left[ \exp(-\alpha_j L_{D1} \xi) |u_j|^2 + 2 \sum_{m \neq j}^M \exp(-\alpha_m L_{D1} \xi) |u_m|^2 \right] u_j \\ (j = 1, 2, 3, \dots, M) \end{aligned} \quad (6)$$

Here,  $T_0$  is the pulse width,  $P_{0j}$  is the incident optical power of the  $j$ th beam, and  $d_{1j}$ , the walk-off parameter between beam 1 and beam  $j$ , describes how fast a given pulse in beam  $j$  passes through the pulse in beam 1. In other words, the walk-off length is

$$L_{W(1j)} = T_0 / |d_{1j}|. \quad (7)$$

So,  $L_{W(1j)}$  is the distance for which the faster moving pulse (say, in beam  $j$ ) completely walked through the slower moving pulse in beam 1. The nonlinear interaction between these two optical pulses ceases to occur after a distance  $L_{W(1j)}$ . For cross phase modulation (CPM) to take effect significantly, the group-velocity mismatch must be held to near zero.

Finding the analytic solution of Eq. (6) which is a set of simultaneous coupled nonlinear Schrodinger equations is a formidable task. However, it may be solved numerically by the split-step Fourier method, which was used successfully earlier to solve the problem of beam propagation in complex fiber structures, such as, the fiber couplers, and to solve the thermal blooming problem for high energy laser beams [6]. According to this method, the solutions may be advanced first using only the nonlinear part of the equations. And then the solutions are allowed to advance using only the linear part of Eq. (6). This forward stepping process is repeated over and over again until the desired destination is reached. The Fourier transform is accomplished numerically via the well-known Fast Fourier Transform Technique.

### III. Soliton on a Single Beam

It is well known that, for an idealized fiber with no loss, optical soliton on a single wavelength beam takes the initial form [1,5]:

$$u(0, \tau) = N \operatorname{sech}(\tau) \quad (8)$$

where  $N$  is the soliton magnitude and

$$N^2 = \frac{L_D}{L_{NL}}. \quad (9)$$

It is also known that the single beam soliton equation is

$$i \frac{\partial u}{\partial \xi} = - \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} - N^2 [|u|^2] u. \quad (10)$$

Here, the dispersion length  $L_D$  and the nonlinear length  $L_{NL}$  are defined earlier in Eq. (6). In the case of anomalous group velocity dispersion (GVD) for soliton,  $\operatorname{sgn}(\beta_2) = -1$ . For the fundamental soliton case,  $N = 1$ . This means that when an initial pulse with pulse shape given by Eq.(8) with an amplitude of unity is launched inside an ideal lossless fiber, the pulse will retain its hyperbolic secant shape without degradation for arbitrarily long distances. One notes that the delicate balance between the dispersion effect

represented by  $L_D$  and the nonlinear self phase modulation effect represented by  $L_{NL}$  occurs at  $N = 1$  for the fundamental soliton. The nonlinear effect on a pulse for a single wavelength beam is embodied in  $L_{NL}$ , while the dispersion effect on the pulse is embodied in  $L_D$ .

## VI. Solitons on Wavelength Division Multiplexed Beams

It is of interest to learn whether solitons exist on WDM beams in a fiber. Starting with an idealized fiber which is lossless (i.e.,  $\alpha_j = 0$  for all beams) and which possesses uniform group velocity dispersion (i.e.,  $v_{gj} = v_g$  for all beams) within the wavelength range under investigation, the equations governing the propagation characteristics of signal pulses are:

$$i \frac{\partial u_j}{\partial \xi} = - \frac{1}{2} \frac{\partial^2 u_j}{\partial \tau^2} - \frac{L_D}{L_{NL}} \left[ |u_j|^2 + 2 \sum_{m \neq j}^M |u_m|^2 \right] u_j$$

$$(j = 1, 2, 3, \dots, M). \quad (11)$$

The anomalous GVD case in which  $\text{sgn}(\beta_{2j}) = -1$ , is considered. It is seen from the above equation that the summation term in the bracket representing the cross phase modulation (CPM) effect is twice as effective as the self phase modulation



(SPM) effect for the same intensity. This observation also provides the idea that cross phase modulation may be used in conjunction with self phase modulation on the WDM pulses to counter act the GVD effect, thus producing WDM solitons. Comparing the bracketed terms in Eqs.(10) and (11) shows that if one chooses the correct amplitudes for the initial pulses on WDM beams and retains the hyperbolic secant pulse form, it may be possible to construct a set of initial pulses which will propagate in the same manner as the single soliton pulse case, i.e., undistorted and without change in shape for arbitrarily long distances. Let us choose the initial pulses as follows:

$$u_j(0,\tau) = (1 + 2(M-1))^{-1/2} \text{sech}(\tau) \quad (12)$$

$$(j = 1, 2, 3, \dots, M),$$

where M is the number of WDM beams.

Using these initial pulse forms numerical simulation was carried out to solve Eq. (11). The split-step Fourier method was used. The fiber parameters used for the simulation are:

$$L = \text{length of fiber} = 1000 \text{ km}$$

$$\beta_2 = \text{dispersion coefficient} = -2 \text{ ps}^2/\text{km}$$

$\gamma$  = nonlinear index coefficient =  $20 \text{ W}^{-1}\text{km}^{-1}$

$T_0$  = pulse width = 10 ps.

$L_D$  = 50 km

$L_{NL}$  = 50 km.

Four cases with  $M = 1, 2, 3, 4$  were treated. The  $M = 1$  case corresponds to the well known single soliton case; here, the amplitude for the fundamental soliton is 1. For the 2-beam case, the amplitude is  $(3)^{-1/2} = 0.57735$ . For the 3-beam case, it is  $(5)^{-1/2} = 0.4472136$ . For the 4-beam case, it is  $(7)^{-1/2} = 0.37796447$ . It is noted that the amplitude of the fundamental solitons on WDM multi-beams becomes successively smaller as the number of beams is increased. This is because the nonlinear effect becomes more pronounced when more beams are present. Numerical simulation shows that after propagating 1000 km through this fiber the original pulse shape for all these WDM pulses remains unchanged. It thus appears that the initial forms chosen for the pulses on WDM beams are the correct soliton forms for WDM beams.

## V. Conclusion

The existence of optical solitons on wavelength division multiplexed beams in a fiber is not only of fundamental interest but also provides enormous implication in the field of optical fiber communications. It is

conceivable that multi-tera bits of information can be sent through a single fiber in the bit-parallel wavelength division multiplexed format [2] without degradation.

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